An Optimization Approach Applied to Fair Division Transportation Funding Allocation Models

by Carlos M. Chang and Edith Montes

The problem of multiple necessities and limited funds is common in the transportation field. Funding allocation for a transportation agency often involves prioritizing the allocation of funds across a number of participants who have their own needs and preferences. If a participant believes that the final allocation is unfair, then this perception could result in the generation of envy. In this paper, a genetic optimization technique is applied to a Fair Division Transportation Funding Allocation Model (FDTFAM) to minimize the total envy based on the participant's own priorities and the budget constraints.

INTRODUCTION

The problem of distributing limited funds across a number of participants with their own priorities is a common management problem. This dilemma has always been an issue for decision makers when allocating funds. The funding allocation problem is usually addressed by decision makers using expert judgment, which involves either subjective criteria, weighted formulas with pre-established priorities, or a combination of both. To obtain consensus on the funding allocation criteria is very difficult due to multiple interests and different perspectives from each participant requesting the funds.

Traditional transportation funding allocation methods generally use formulas based on population and number of highway miles managed by the participant. However, these methods may result in more funding to larger cities or districts. This outcome may be perceived as unfair by smaller cities or districts. This perception regarding the final allocation of funds could result in the generation of envy. Envy is felt by a participant if he or she perceives that less funding is received when compared with other participants. In terms of financial allocation, envy is the expression of an unbalanced distribution of funds affecting the overall economic growth in the region with the addition of social discomfort. This paper provides an innovative funding allocation approach based on fair division methods. The approach considers individual preferences from the participants to prioritize the projects requested for funding. The objective is to minimize the envy received by each participant under budget constraints. The mathematical formulation of envy and the funding allocation model used to minimize the total envy, and the application of an optimization genetic technique to solve the model are major contributions of this research.

Envy Definition

Envy is expressed in terms of the differences among allocated to requested ratios of each participant. The higher the envy, the higher the difference between each participant's ratios of allocated to requested funds. These ratios represent the percentage that each participant received with respect to total requested funds; i.e., a 1.0 ratio means that the participant received 100% of the requested funds and there is no envy felt by this participant.

FDTFAM uses Equation 1 to calculate the envy felt by the i^{th} participant with respect to the j^{th} participant. For instance, if the i^{th} participant received less than the j^{th} participant, then participant i

will feel envy of participant *j*. However, if participant *i* received more than participant *j*, no envy is felt from *i* to *j*.

(1) $\varepsilon_{ij} = \{|\rho_i - \rho_j|$

where:
ρ_i = Allocated to requested funding ratio of i^{th} participant
ρ_j = Allocated to requested funding ratio of j^{th} participant
ε_{ij} = envy perceived by the i^{th} with respect to the j^{th} participant only if $(\rho_i - \rho_j) < 0$, 0 otherwise

FUNDING ALLOCATION METHODS IN TRANSPORTATION PROJECTS

There are different methods to approach funding allocation. Traditional methods include formulas set by a given agency to prioritize projects requested for funding. These formulas assign weights to performance measures or indicators to determine the level of priority for each individual project. However, these methods may lead to the public's disagreement if decisions are perceived as unfair or not equitable. On the other hand, optimization methods are meant to maximize or minimize a given objective. Generally, these methods are used to maximize benefits with budget constraints or to minimize costs subjected to certain constraints. Optimization methods aim to provide optimal solutions to a given problem.

Chan et al. (2003) suggests a genetic-algorithm (GA) optimization solving technique to allocate the total funds available to the district agencies in order to achieve target performance. The funding allocation problem considers the overall objective of the agency as well as the district target objectives by using a two-stage algorithm. The first stage uses the regional or district objectives and constraints to generate possible solutions to the problem. Then, the second stage analyzes the solutions obtained in the first stage using the objectives and constraints of the individual agencies. The results of the proposed technique is shown in terms of the network pavement condition and compared to the results of typical funding allocation methods. This paper shows an innovative solution method; however, the two-stage algorithm cannot be used to solve multi-objective problems while solving all objectives simultaneously. Tsunokawa and Van Hiep (2008) presented an optimization approach for the allocation of a system-wide budget among road assets. This method uses the net present value (NPV) as the common denominator for prioritizing the funding allocation in all asset subsystems. Using an asset subsystem optimizer (ASSO), the NPVs are used to generate the NPV functions to find the optimal allocation among all subsystems. Once the optimal budget allocation is obtained, the ASSO is used to find the optimal management strategy using the optimally allocated budget.

Most recently, innovative funding allocation methods have been used to maximize the benefits of funds invested in projects. Despite the method used for maximizing funding allocation, there is a pre-established criteria that applies to all the participants when setting priorities for project selection. Individual preferences of participants may be based on a different criteria based on their own perspective and local needs. This paper presents a fair division funding allocation transportation model (FDTFAM) to incorporate individual preferences in the decision-making process. The model is based on fair division concepts of proportionality, envy-freeness, equitability, and efficiency.

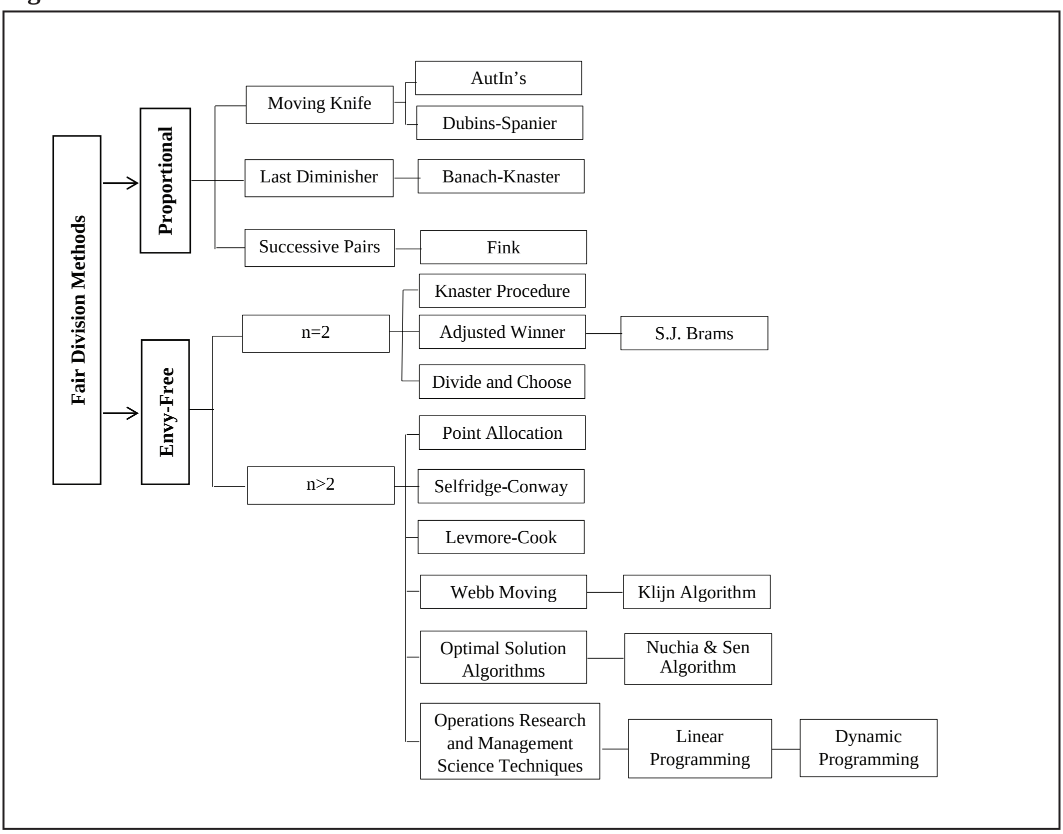
LITERATURE REVIEW ON ENVY AND FAIR ALLOCATION METHODS

The problem of dividing resources fairly can be generalized by defining the allocation of a resource over *n* number of participants expecting to receive a portion of the resource. In order to achieve fair division, it is desired that the procedure implemented satisfies four requirements. First, the procedure must lead to a proportional distribution, i.e., each participant expects to receive at least $1/n$ of the resource. Second, the procedure must be envy-free, i.e., each participant believes that the received amount is fair and there is no reason to exchange their share. Third, the procedure must be equitable; i.e., individual valuation of the portion received by one participant is equal to the valuation of the other participants. Fourth, the procedure must be efficient or Pareto optimal, i.e., no other allocation would benefit one participant without affecting another *(*Nuchia and Sen 2001*)*. The achievement of these four fair division characteristics simultaneously could be guaranteed only for two participants (Dupuis-Roy and Gosselin 2009). In most real problems, there are more than two participants, and it is very unlikely in practice to fully achieve proportional, envy-free, equitable, and efficient solutions (Brams and Taylor 1996).

There are several fair division theoretical methods that attempt to solve the problem of dividing any type of resource among several participants while trying to achieve a proportional, equitable, fair, and envy-free allocation for all. These methods have been used to achieve a fair allocation of divisible and indivisible goods, such as the Divide and Choose Procedure (Barbanel and Brams 2004), the Moving Knife (Barbanel and Brams 2004), the Last-Diminisher “Trimming Algorithm” (Austin 1982), the Successive Pairs Algorithm (Austin 1982), the Knaster’s Procedure (Brams and Taylor 1996), the Adjusted Winner (AW) Procedure (Brams and Taylor 1993), and the Point Allocation (Saunders 2011).

All fair division methodologies strive to make allocations based on two main characteristics: proportionality or envy-freeness. Therefore, fair allocation methods can be classified into two groups: proportional and envy-free methods. A proportional allocation method attempts to assign the funds in a manner that all the participants receive the same amount. An envy-free allocation method strives to distribute the items based on the participants’ preferences; funds are assigned to the participant who shows more desire for it. Figure 1 shows an overview diagram of current fair division methods (Chang et al. 2011).

Figure 1: Overview of Fair Division Allocation Methods



Proportional Methods

The Moving Knife. The moving knife procedure is inspired and illustrated by the process of how to fairly divide a cake among several participants and to satisfy preferences with the fewer amounts of cuts (Barbanel and Brams 2004). To illustrate this procedure, consider that a resource *A* is a rectangular cake of length *X* with constant width. It is to be divided into x_n pieces; where *n* represents the number of participants. In order to divide the cake, one of the participants places a knife on the left side of the cake and perpendicular to the length X. Then, the participant moves the knife continuously to the right of the cake and makes a cut when one of the other players calls for it. It is perceived that the knife has moved a distance x_1 that yields at least 1/*n* of the cake (Barbanel and Brams 2004). After the cut is made, the piece is given to whoever placed the call and then leaves. If two or more persons call for a cut, the piece will be given randomly to one of them. This process continues for *n*-1 remaining participants until one participant is left.

Last-Diminisher "Trimming Algorithm." The Last-Diminisher trimming algorithm method is not continuous. In this procedure, participant 1 cuts a piece of size 1/*n* and participant 2 takes the piece and trims it if he believes that its size is greater than 1/*n*. The piece is passed successively and trimmed until it reaches Participant *n*–1. The participant *n* can take the piece to conclude, otherwise it is allocated to the last person who trimmed it. This process is repeated with the remaining pieces until only one participant is left (Austin 1982).

Successive Pairs Algorithm. In this method, the problem considers that a resource has already been divided among *n* participants and each participant owns at least 1/*n*. Let's assume that a new participant is included in the division of the resource. Now, the resource has to be distributed for *n*+1 participants. Consequently the original participants share has been converted to 1/(*n*-1). Participants 1 through *n*–1 are now required to divide their pieces into *n*+1 equal parts. After this division has been performed, the new participant is allowed to choose one part from participants 1 through 1–*n*. This will guarantee that each of the participants receives at least (*n*)/(*n*+1) from 1/(*n*) (Austin 1982).

Proportional methods assume that the goods are divisible; but in the real world, a combination of divisible and non-divisible goods is very common, e.g., machinery, equipment, and buildings. Proportional methods are useful when the item to be distributed is continuous. The other main limitation of proportional methods is that they only guarantee envy-free allocations in distributions between two participants (Brams and Taylor 1996).

Envy-free Methods

Knaster's Procedure. In Knaster's procedure, there is a set of assets *A* that have to be distributed among *P* participants. The Knaster's procedure resembles an auction because each good is assigned to the highest bidder. The total funds to be allocated is divided among the participants and requires that each individual owns some initial amount of money placed as a deposit. This deposit is used to pay those individuals who receive less or nothing at the end of the bidding as compared to everybody else (Chang et al. 2011).

Adjusted Winner (AW)Procedure. The AW algorithm was proposed by Brams and Taylor (1994) to provide an envy-free, equitable, and efficient solution. The goods are divided as in the Knaster's procedure without the need of a deposit, and then adjusted to make the number of points of each participant equal to each other (i.e., the goods are redistributed to achieve equitability).

Divide and Choose Protocols. The Divide and Choose method where "one divides and the other chooses" is one of the oldest methods. In this method, each participant receives at least 1/*n* of the good in question, where n is the number of participants. The proportion is defined by the participants' own evaluation. Let's consider the problem where two individuals resolve to share a divisible good. This problem can easily be solved in two steps: first, one participant divides the good, and then the second participant chooses the share portion (Barbanel and Brams 2004).

Point Allocation. In point allocation, a hypothetical number of points, e.g., 3, 5, or 10 are used to formulate a preference list by each participant. This allocation is based strictly upon decision makers' judgments. In this procedure, each participant assigns a value to any good in consideration; then, the participant with the highest scores per good obtain the corresponding item. An envy ratio (assigned score to actual score) is calculated to assess the envy-freeness of the allocation (Saunders 2011).

Envy-Free and Optimal Solution Algorithms. Fair division methods aim to achieve envy-freeness in the allocation of goods; however, it does not provide optimal solutions. The development of algorithms that improves envy-free and efficient allocations at the same time has gained more interest. Procedures which contain similar notions to Knaster's have been developed to find envy-free and efficient solutions to the fair division problem. Aragones (1995) proposed an algorithm that finds envy-free solutions where the number of participants is equal to the number of items. The items are a set of indivisible objects and a fix monetary amount. The benefit that the participants receive from the allocation is approximated by a quasi-linear utility function. To be initialized, the algorithm requires a Pareto efficient allocation of the resources. Another algorithm developed to determine envy-free and efficient allocations was proposed by Klijn (2000). This algorithm was developed for resource allocation in the public sector (Foley 1967) to achieve equity, envy, and efficiency, and was based in a derivation of the money Rawlsian solution (Varian 1974). Klijn's algorithm is similar to Aragones's but it does require an initial Pareto allocation of the objects. It is set with a random allocation followed by a directed graph with nodes that correspond to the objects. The vertices of this graph represent the indifference or envy among the participants. The algorithm eliminates the envy vertices in order to provide an envy-free and efficient allocation solution.

Another approach to improve envy-free procedures was proposed by Nuchia and Sen (2001). In their approach, the participants are referred to as agents. A two-stage protocol is used to identify all possible exchanges between the agents in order to improve the efficiency in envy-free allocation procedures. A graph $G = (V, E)$ is defined where the vertices represent the agents. Each vertex is assigned a weight representing the net gain in utilities from a possible envy-free exchange. The exchange is possible only if both agents have a gain in utilities. The iteration of the protocol continues until no additional improvement can be achieved.

Application of Optimization Solving Techniques to Fair Division Methods. Fair division methods are difficult to solve when trying to achieve a perfect envy-free result. Different optimization techniques could be applied as a solving technique to fair division problems. For example, a linear programming approach using the concepts of side payments was developed by Kuhn (1967). In 2007, the problem of allocating indivisible objects between two participants was formulated as an Integer Linear Programming problem in Dall'Aglio and Mosca (2007). In this technique, dynamic programming is used in combination with a branch and bound technique (Adjusted Winner procedure) to find an optimal solution using the backtracking procedure of the knapsack algorithm.

The common fair division problem tends to be NP-complete (Nondeterministic Polynomial) and desires to minimize envy (Vetschera 2010). There are no fast solutions known for NP-complete problems. However, there are several approaches to achieve a fair division of goods minimizing envy among participants. For example, Lipton et al. (2004) focus on setting an upper bound to minimize envy in which the bound is determined by a utility function. Nevertheless, this algorithm is only useful when the participants' utility functions are the same. Rudolph Vetschera (2010) considered the fair division problem with two participants for a set of indivisible items and applied a branch and bound technique to solve the problem. In this approach, the bounds ignore the indivisibility of the items and the different participants' valuations.

As we have described, the literature review on fair division methods is very broad but focused mainly on theory and abstract examples. In a real situation, there are usually more than two participants requesting funds and competing for indivisible projects. Therefore, it is very unlikely to fully achieve simultaneously the four fair division characteristics of proportional, envy-free, equitable, and efficient solutions when there are several participants requesting funds. In spite of this limitation, the application of fair division methods strive to make allocations among several participants focusing mainly on proportionality and envy-freeness.

THE FAIR DIVISION TRANSPORTATION FUNDING ALLOCATION MODEL

Fair division models aim to provide a fair share of the resource based on the own participant's valuation of the resource; therefore, they will not envy the others. Envy occurs when a participant feels that the share received is unfair when compared with the others. Börgers (2010) and Brams & Taylor (1996) describe that a fair share of a particular participant could be represented as his/her own valuation of the resource divided by the total number of participants.

The Fair Division Transportation Funding Allocation Model (FDTFAM) combines concepts of the Point Allocation and Adjusted Winner (AW) methods. FDTFAM applies the Point Allocation concepts to define the preferences of the participants. Preference or desirability of a project represents how desirable a certain project is to the participant based on his/her own criteria, and it is expressed by distributing a total of 100 points among the projects in the participant's wish list (Chang et al. 2011). The concepts of the AW method are used to minimize the envy. The optimum solution of the FDTFAM looks for envy-free, equitable, and efficient allocations taking into consideration the participant's preference or desirability.

FDTFAM Mathematical Formulation

FDTFAM aims to minimize total envy and to maximize the participants' desirability subjected to budget constraints. Desirability represents how desirable a certain project is to the participant. The mathematical model of the FDTFAM is shown in Equations 2 through 6. These equations were inspired by the envy definitions (Brams and Taylor 1996) and the point allocation method (Saunders 2011).

$$(2) \quad \text{Maximize } D : \sum_{i=1}^{N} P_{ik} X_{ik}$$

$$(3) \quad \text{Minimize } E : \sum_{i=1}^{N} \varepsilon_i X_{ik}$$

Subject to:

$$(4) \quad \sum_{i=1}^{N} C_{ik} X_{ik} \leq b \qquad k = 1, 2, 3, .. m$$

$$(5) \quad \sum_{k=1}^{m} X_{ik} \geq 1 \qquad \forall i = 1,2,...,n$$

$$(6) \quad X_{ik} \in \{0,1\}$$

Where:

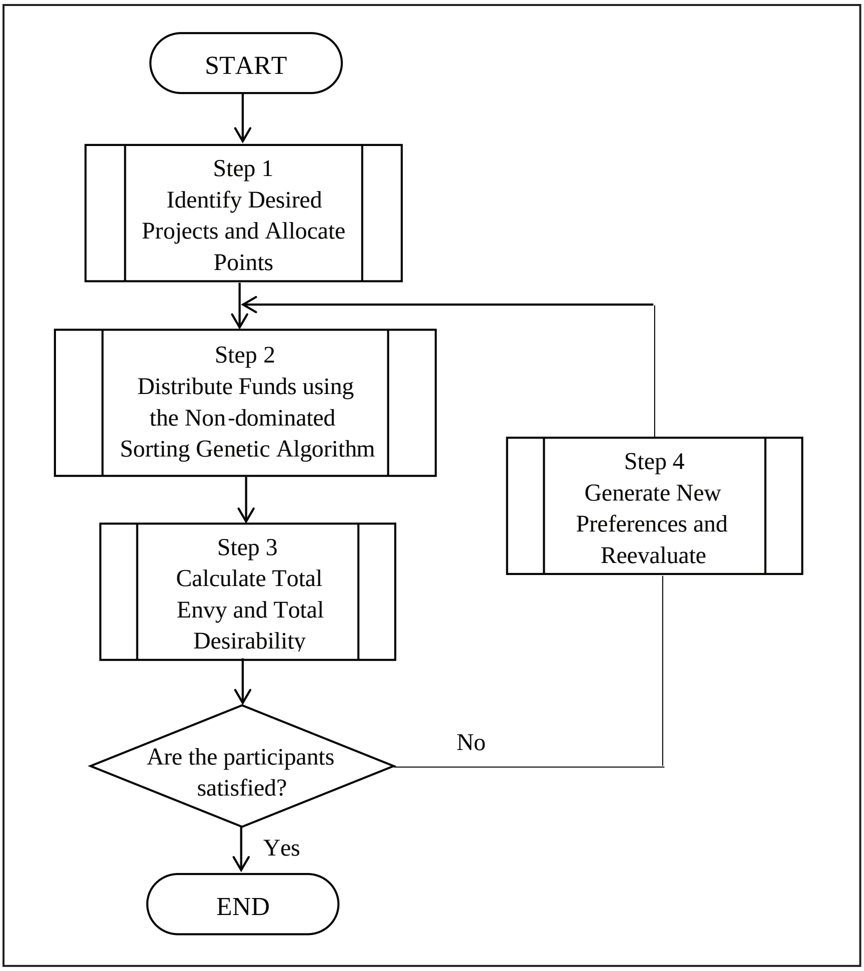
E = objective function for minimizing total envy
D = objective function for maximizing total desirability of projects selected for funding
ε_i = envy perceived by the i^{th} participant
P_{ik} = desirability assigned by participant i to project k
X_{ik} = 1 if participant i requested project k and funds are allocated; 0 otherwise
C_{ik} = cost assigned by participant i to project k
b = total budget available
i = represents each participant
N = total number of participants
k = represents each project requested by the participant
m = total number of projects requested by a participant

FDTFAM will result in a low overall envy by selecting projects with high desirability based on the participant's individual preferences. Participants are given 100 points to distribute among their wish lists of projects to express their desirability or preferences. The points assigned to a project measure the desirability or preference that a participant has for that project. Higher desirability is expressed by a higher assignation of points. In this method, each participant follows his/her own criteria to prioritize the projects requested for funding.

Funding Allocation and Total Envy

Envy ratios are calculated in the process and added together to calculate the total envy produced by the final allocation of funds. Also, the total desirability is calculated by adding the preference points that each participant obtained at the end of the funding allocation process. If the total envy due to the allocation is equal to zero, or the participants are satisfied with the allocated funds, then the process stops; if not, another wish list of ranked projects with assigned points is requested of the participants. In practice, it is very unlikely to achieve a total envy equal to zero when allocation involves more than two participants. Therefore, the process is repeated until all the participants are satisfied, or envy is minimized, with the final allocation. Figure 2 shows a flow chart summarizing the steps of the funding allocation process.

Figure 2: Step by Step Fair Division Transportation Funding Allocation Process (Source: Chang et al. 2011)



Genetic Solving Technique for the FDTFAM

The mathematical formulation of FDTFAM is solved using a Non-dominated Sorting Genetic Algorithm (NSGA-II). Genetic algorithms provide solutions to multi-objective problems, and are able to search for multiple solutions, thus preventing a local optimum. This algorithm uses an evolutionary process (natural selection) with substitutes for evolutionary operators, including selection, genetic crossover, and genetic mutation. In the selection process, only the best adapted survive; in this case, the answers that best accomplish the goals are recorded and used as possible solutions. The genetic crossover resembles the chromosome interchange taking place in evolution, possible individual solutions are interchanged in random sets of alternative solutions. The genetic mutation resembles a random mutation in an individual; in this case, it leads to a different solution from a random set of solutions. The population is sorted into a hierarchy of sub-populations based on the ordering of Pareto dominance. A solution is Pareto dominated if the solution increases the allocations of one participant while making the other participants' allocations decrease. Once the population is sorted, the members of each sub-group is evaluated, and the resulting groups and similarities are used to promote a diverse set of non-dominated solutions. NSGA varies from a simple genetic algorithm only in the manner that the selection operator works. The crossover and mutation operators remain as usual. Before the selection is performed, the population is ranked on the basis of an individual's non-domination. A solution is non-dominated if it improves one objective without affecting the other objectives (Srinivas and Kalyanmoy 1994).

The NSGA-II was used to solve the FDTFAM because it is part of the genetic algorithms. As previously mentioned, this algorithm is recommended for the funding allocation problem because it searches for multiple combinations of solutions by using crossovers and mutations among sets of solutions or populations to guarantee that the solution is not just a local optimum.

CASE STUDY

A case study is presented in this paper to show the application of FDTFAM to allocate funds among competing projects. FDTFAM can be used by departments of transportation (DOTs), metropolitan planning organizations (MPOs), or any other agency. The methodology was tested by the Texas Department of Transportation (TxDOT) using real data and a number of case scenarios (Chang et al. 2013).

In the case study, 10 participants are competing for funding. It is assumed that the participants are MPOs with different population sizes under their jurisdictions. An MPO is a policy-making organization made up of local government representatives and governmental transportation authorities for urbanized areas with populations greater than 50,000. Small MPOs will have a population of 50,000 to 200,000; medium MPOs a population of 200,001 to 1,000,000; and large MPOs a population greater than 1,000,000. Table 1 shows the population and size of each participant in the case study.

Table 1: Population and Size Assignation of Participants

Participant	Population	Size
1	86,793	S
2	120,877	S
3	194,851	S
4	405,027	M
5	405,300	M
6	800,647	M
7	1,714,773	L
8	1,716,289	L
9	6,087,133	L
10	6,539,950	L

Since FDTFAM considers each participant's project preference without requesting their individual criterion to define the priorities, the model is not directly affected by the project location. The number of requested projects, the desirability assigned to each project, and budget constraints are factors that will influence the funding allocation results.

Table 2 describes the five different scenarios used to allocate funds among the projects requested by the 10 participants. The manner that points were assigned to the desired projects by the participants varies in each scenario.

Table 2: Project Funding Allocation Scenarios

Scenario	Description
1	All participants assigned preference points to all projects in a range of: 1-15
2	Participant 8 (large region) gave 86 points to the highest cost project, and assigned 1 point to each of the remaining projects. The rest of the participants have same points per projects per category as in scenario 1(point's range: 1-15)
3	Points based on percentage of project cost with respect to total requested
4	Each participant has a different total number of points based on population. The total number of points for each participant was calculated based on the population served. Participant 1 has the smallest population served (86,793) and participant 10 has the largest (6,539,950). So, while participant 1 gets the baseline 100 points, participant 10 gets 7,535 population-based points because it is 75.35 times more populous than participant 1
5	Participant 8 (large region) asks for 15 projects while the rest of the participants only ask for 10 projects. Participant 8 tries to get more funding by increasing the total expected funds

The scenarios were developed to reproduce current practices observed in the recent past. In Scenario 1 the participants show their true preferences. In Scenario 2, one participant tries to manipulate the outcomes to obtain one project funded by assigning almost all of its points to that project. In Scenario 3, participants prioritize their project preferences based on the project costs. In Scenario 4, participants distribute their points among projects based on population. In Scenario 5, a participant tries to manipulate the outcome by requesting more projects than really needed in an attempt to obtain more funds.

To solve this multi-objective optimization problem, SolveXL software was selected due to its technical capabilities, ease of use, and ability to work with Excel spreadsheets. SolveXL is an add-in for Microsoft Excel, which uses evolutionary (genetic) algorithms to solve complex optimization problems. The application is written in C++ and opens an interface to interact with Microsoft Excel. It has a user friendly Wizard with built-in help that allows the user to configure the tool easily and to perform optimization using regular Excel formulas. The model can be set up in the same way as the Excel Solver. SolveXL is able to solve many types of single- and multiple-objective problems using genetic algorithms (SolveXL 2013).

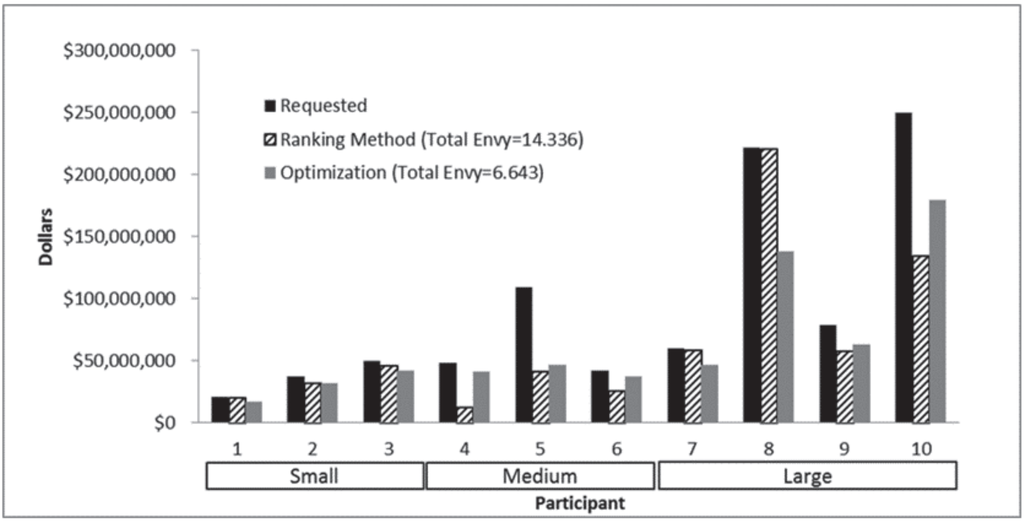
To compare the results obtained by the optimization method, the case study was also solved with a ranking method developed for the Texas Department of Transportation (TxDOT) (Chang et al. 2013). In the ranking method, the projects are ranked from the highest to the lowest desirability based on the distribution of points provided by each participant. The project with the highest desirability is selected for funding first. The second project in the ranked list is "bubble up" to the top of list and selected for funding if money is available. Projects in the wish list continue to "bubble up" for funding until the budget is exhausted, or the ranked list of projects ends. This method is called the Dynamic Bubble Up technique (DBU) because it "bubbles up" projects from the bottom to the top for funding (Chang 2007).

All five scenarios were solved with FDTFAM using the ranking (bubble up technique) and the optimization solving technique for the purpose of comparison. The results obtained in each scenario are summarized as follows.

Scenario 1

In scenario 1, all participants assigned preference points to all projects in a range of 1-15. The results show that the optimization method resulted in a lower total envy of 6.643 as compared with the ranking method, which obtained a total envy of 14.336. Participants that received almost all the requested funds in the ranking method received less funds in the optimization method as shown in Figure 3. It is observed that the small participants received higher allocated amounts of funding, which were very close to the requested funds. This indicates that this group of participants will have the smallest envy. On the other hand, the medium sized participants observed the highest envy. This group had the highest envy because participant 5 requested very high cost projects. The total allocated funds to medium sized participants were even higher than two of the large participants.

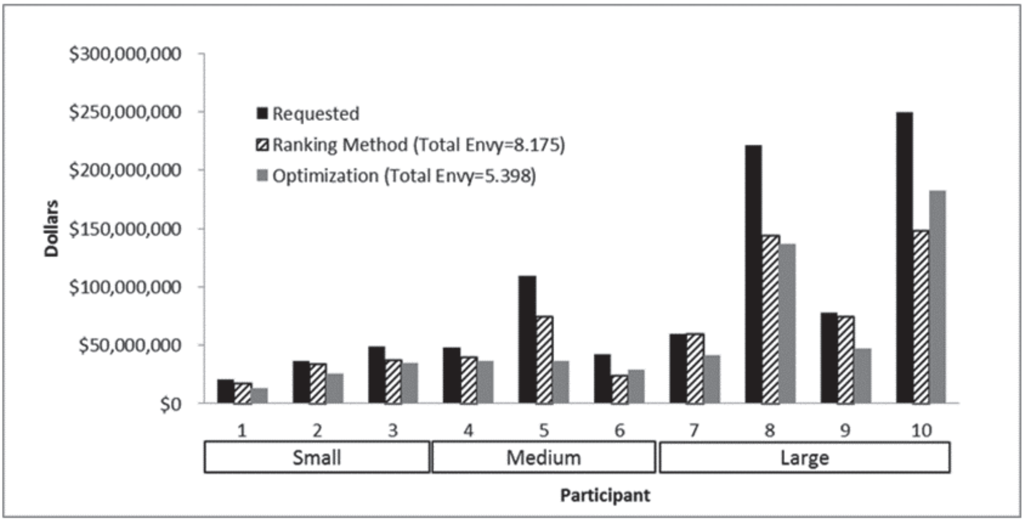
Figure 3: Comparison of Funding Allocation Results in Scenario 1



Scenario 2

In scenario 2, participant 8 (large) assigned its 86 points to the highest cost project, and 1 point to each of the remaining 14 projects. The rest of the participants have the same points per projects as in scenario 1 (points range: 1-15). This scenario simulates a situation in which a participant tries to manipulate the allocation in his/her own favor by assigning very high preference to one project. In this scenario, the optimization method resulted in a lower total envy of 5.398 as compared with the ranking method, which obtained a total envy of 8.175. Participant 8 obtained lower total funds because it assigned very high preference points (86 pts.) to one project. Using the ranking method, participant 8 obtained $143,800,494 in scenario 2 and $220,437,683 in scenario 1. In the optimization method, it received $136,741,309 in scenario 2 and $137,772,553 in scenario 1. These results are shown in Figures 3 and 4. Participant 8 did receive funding for the project with the highest preference points because the desirability of this project was significantly higher than the other projects but the total funds were lower. In this scenario, the medium-sized participants obtained the highest envy ratios.

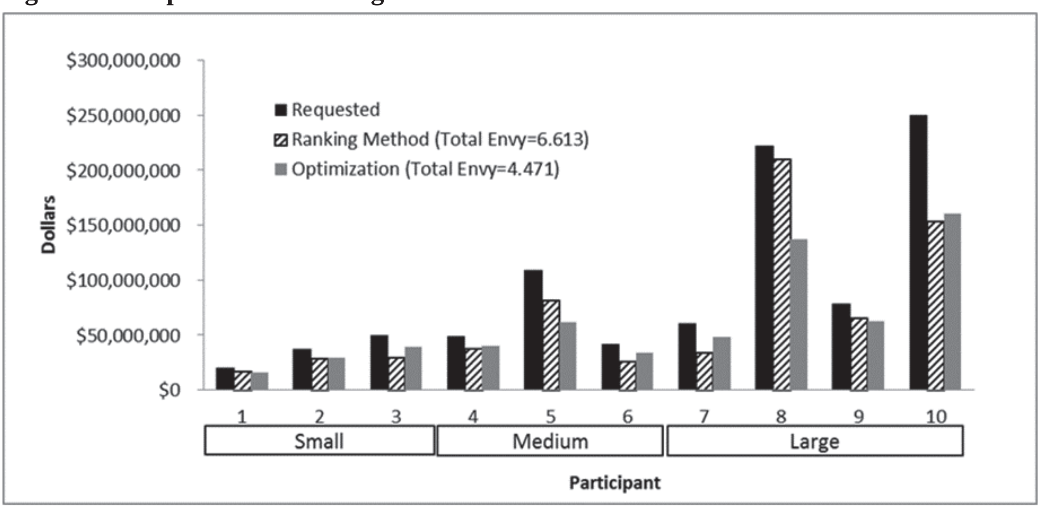
Figure 4: Comparison of Funding Allocation Results in Scenario 2



Scenario 3

In scenario 3, preference points were based on the percentage of the project cost with respect to total funds requested by the participant. The results show that the optimization method resulted in a lower total envy of 4.471 as compared with the ranking method with a total envy of 6.613. This scenario shows the lowest total envy as compared with all the other scenarios. In this scenario, the participant's preference points are based on the project cost. Projects with higher costs were assigned a higher desirability, resulting in higher allocated/requested ratios. Funding allocation results of scenario 3 are shown in Figure 4. The participants with the highest envy were the large-sized participants.

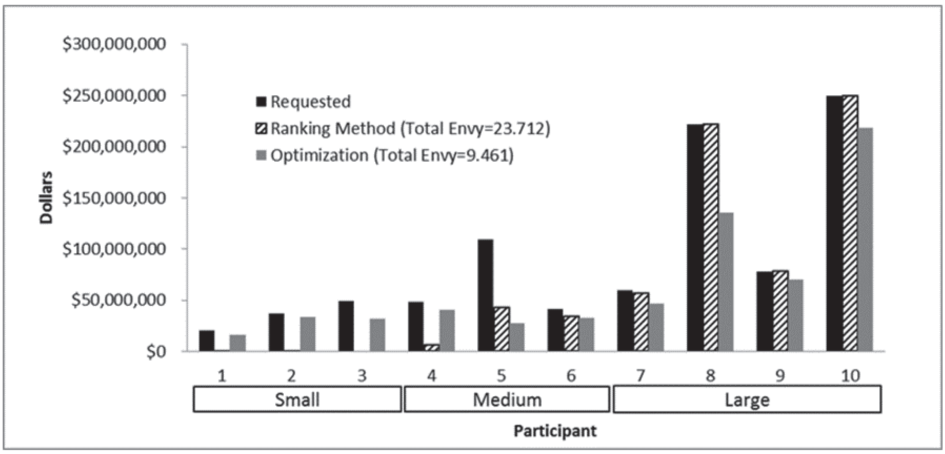
Figure 5: Comparison of Funding Allocation Results in Scenario 3



Scenario 4

In scenario 4, each participant had a different total number of points based on population. The total number of points for each participant was calculated based on the population under their jurisdiction. Participant 1 had the smallest population (86,793) and participant 10 had the largest population (6,539,950). For instance, while participant 1 gets the baseline 100 points, participant 10 gets 7,535 population-based points. The optimization method resulted in a total envy of 9.461 and 23.712 with the ranking method. The distribution of points based on the population resulted in a higher total envy because smaller participants received fewer preference points to distribute among projects. Figure 6 shows that large participants obtained almost all their requested funds and the small participants received almost none of the requested funding. This resulted in the highest total envy when compared with other scenarios.

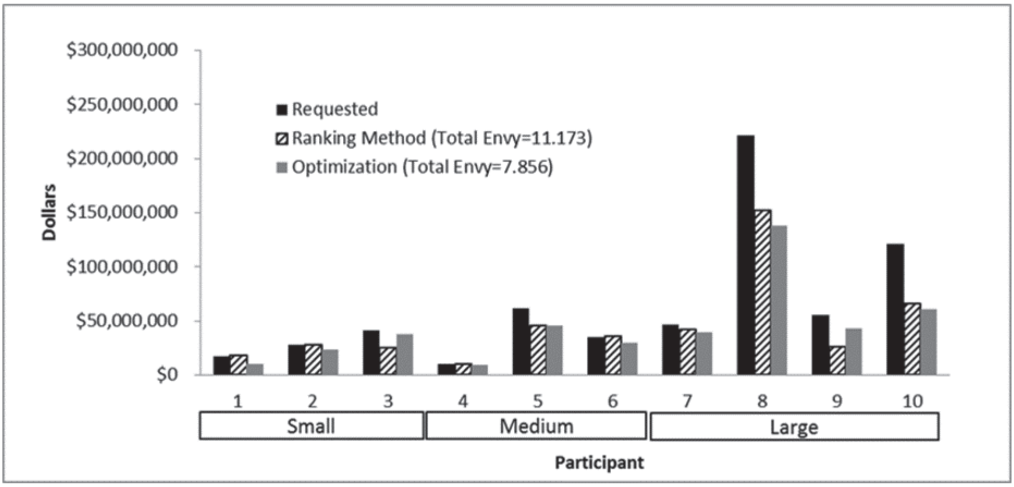
Figure 6: Comparison of Funding Allocation Results in Scenario 4



Scenario 5

In scenario 5, participant 8 (large) requested 15 projects while the rest of the participants only asked for 10 projects. Participant 8 increased the number of projects in an attempt to obtain more funding by increasing the total amount of requested funds. Figure 7 shows that the optimization method resulted in a total envy of 7.856 and the ranking method had 11.173 of total envy. Using the ranking method, participant 8 obtained $152,087,683 of $221,587,683 as shown in Figure 6. This represents a lower allocation when compared with scenario 1 where the same participant obtained $220,437,683 of $221,587,683. Participant 8 did not obtain extra funding due to the lower points assigned to the projects requested for funding. The largest difference between allocated to requested amounts occurred for large participants (Figure 7). The low allocated/requested ratios for large participants demonstrate that the model cannot be manipulated by increasing the number of projects to obtain more funds. In this scenario, participant 8 requested more projects than the others but had to distribute its 100 points over a larger number of projects than the other participants. This low allocation of points is interpreted by the model as low preference; therefore, no extra funding was allocated for those projects.

Figure 7: Comparison of Funding Allocation Results in Scenario 5



Ranking Versus Optimization as a Solving Technique

Figures 3 through 7 showed the total funds allocated per participant. The results from the ranking and from the optimization methods were compared for each scenario. It was observed that the optimization method did minimize the envy and maximized the project's desirability. The optimization method provided better results from a fair perspective approach than the ranking method.

Figures 8 and 9 show the results obtained in the ranking and optimization methods per group of participants (small, medium, large). It is observed that total envy is increased in the groups with the largest disadvantage in terms of size, either population or highway miles; for example, participants with a small population or low highway miles. Figure 8 shows that Scenario 4 results in the highest total envy (23.712) since large participants received more funding than small participants. On the other hand, the scenario with the lowest total envy was scenario 3 (6.613) in which desirability was assigned based on the cost of the projects as compared with the total funds requested per participant.

Figure 8: Total Envy per Scenario Using the Ranking Method

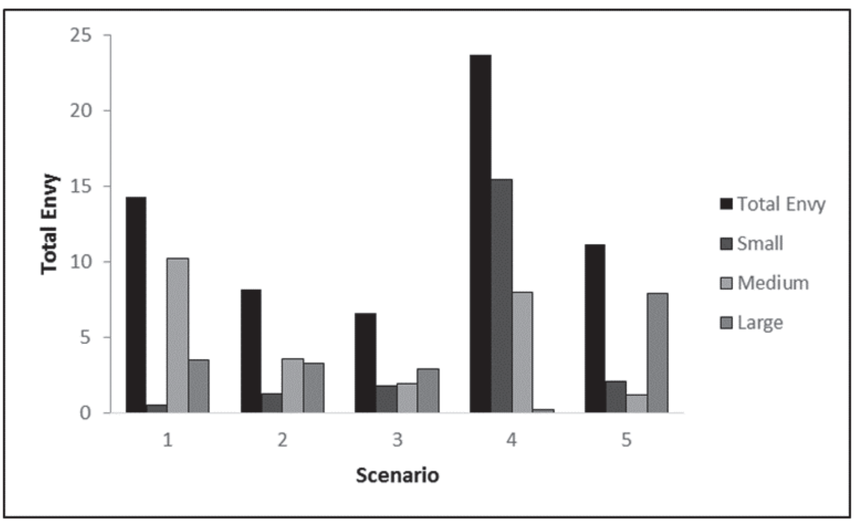
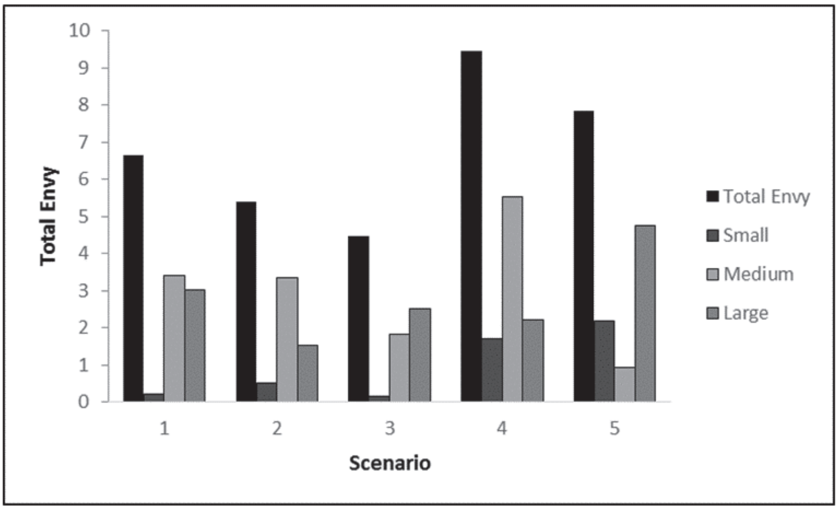


Figure 9 also shows that scenario 4 resulted in the highest total envy (9.461) while scenario 3 in the lowest total envy (4.471). It is worth mentioning that even though the ranking and optimization methods found the same scenarios for the highest and the lowest envy, the selected projects were different. These differences in project selection are caused by the approach used to solve the funding allocation problem. The ranking method maximizes desirability while the optimization method maximizes desirability and minimizes envy at the same time. The projects selected for funding using the optimization method correspond to the optimal solution to the funding allocation problem.

Figure 9: Total Envy per Scenario Using the Optimization Method



CONCLUDING REMARKS

The FDTFAM uses fair division concepts to minimize envy and to maximize the participant's desirability of projects requested for funding. Due to its flexibility, FDTFAM can be applied by agencies in a project-to-project based selection resulting in the highest desirable allocation of funds, and the lowest total envy as perceived by the participants.

FDTFAM can be solved using a Dynamic Bubble-Up Technique, which is based on the ranking method, or an optimization genetic algorithm method. The ranking method strives to maximize desirability per scenario while the optimization method aims to minimize envy and maximize desirability simultaneously. It was observed from the results of the case study that the optimization method provides more even allocated/requested ratios in the final project selection. The smaller differences for allocated/requested ratios among participants result in a lower total envy when compared with the ranking method.

It is also noted that one participant cannot manipulate the fairness of the allocation process. In the absence of knowing the other participant's preferences, attempts at manipulation can result in lower funds allocated to this participant. If a participant tries to trick the allocation process by assigning very high points to only one project or by requesting more funds than really needed, it results in lower total funds allocated to this participant and consequently a higher envy when compared with the other participants.

FDTFAM can be applied at different management levels and in any area of interest in a DOT. The major advantage of FDTFAM is that the individual preferences of each participant are taken into consideration in the funding allocation process. Therefore, the application of FDTFAM to allocate funds among competing projects will result in more proportional, equitable, efficient, and envy-free allocations as perceived by the participants requesting funds. In this sense, FDTFAM is an alternative approach to traditional methods based on pre-established formulas for prioritizing funding allocations. The implementation of FDTFAM in the funding allocation process will provide decision makers with first-hand information about the participants' project priorities and budget needs. This will result in more defensible project funding allocations when justifying formulated budgets.

Acknowledgments

The authors would like to express their sincere appreciation to the Texas Department of Transportation (TxDOT) for providing funding to conduct research project 0-6727 "Using Fair Division Methods for Allocating Transportation Funds." We want to acknowledge the Project Manager Cary Choate, and the Project Monitoring Committee members, including John Ibarra, John Sabala, Michelle Veale, Sara Bagwell, Robert Ramirez, and Teresa Lemons for their support. Special thanks to Ron Hagquist, who introduced this research topic to TxDOT. We also acknowledge the contributions of Dr. Heidi Taboada and Dr. Jose Espiritu at UTEP during the development of FDTFAM.

References

Aragones, E. "A Derivation of the Money Rawlsian Solution." *Social Choice and Welfare 12*, (1995): 267-276 .

Austin, A. K. "Sharing a Cake." *Mathematical Gazette* 66 (437), (1982): 212-15.

Barbanel, J. B. and S. J. Brams. "Cake Division with Minimal Cuts: Envy-free Procedures for 3 Persons, 4 Persons and Beyond." *Mathematical Social Sciences* 48, (2004): 251-269.

Brams, S. J. and A.D. Taylor. "Fair Division by Point Allocation." C.V. Starr Center for Applied Economics, New York University, (1993), RR#92-43.

Brams, S. J. and A.D. Taylor. *Fair Division: from Cake-Cutting to Dispute Resolution*. Cambridge University Press, New York, 1996.

Börgers, C. "Mathematics of Social Choice, Voting, Compensation and Division." Society for Industrial and Applied Mathematics, 2010.

Chan, W.T., T.F. Fwa, and J.Y. Tan. "Optimal Fund-Allocation Analysis for Multidistrict Highway Agencies." *Journal of Infrastructure Systems* 9 (4), (2003): 167–175.

Chang-Albitres, C. "Development of a Multi-Objective Strategic Management Approach to Improve Decisions for Pavement Management Practices in Local Agencies." Ph.D. Dissertation, Texas A&M, College Station, Texas, 2007.

Chang-Albitres, C., H.Taboada, J. Espiritu, and T. Fullerton. "Research Project 0-6727: Using Fair Division Methods for Allocating Transportation Funds." University of Texas at El Paso (UTEP), El Paso, Texas, 2011.

Chang-Albitres, C., H. Taboada, J. Espiritu, T. Fullerton, E. Montes, and A. Walke. "Using Fair Division Methods for Allocating Transportation Funds." University of Texas at El Paso, Final Report Research Project 0-6727, El Paso, Texas, 2013.

Dall'Aglio M. and R. Mosca. "How to Allocate Hard Candies Fairly." *Mathematical Social Sciences* 54, (2007): 218-237.

Dupuis-Roy, N. and F. Gosselin. "An Empirical Evaluation of Fair-Division Algorithms." Proceedings of the 31st Annual Conference of the *Cognitive Science Journal,* (2009): 2681-2686.

Foley D. K. "Resource Allocation and the Public Sector." *YALE Economic Essays* 7 (1), (1967): 45-98.

Klijn, F. "An Algorithm for Envy-free Allocations in an Economy with Indivisible Objects and Money." *Social Choice and Welfare* 17, (2000): 201-216.

Kuhn, H. W. "On Games of Fair Division." *Essays in Mathematical Economics in Honor of Oskar Morgenstern*. Princeton University Press, 1967.

Lipton R., E. Markakis, E. Mossel, and A. Saberi. "On Approximately Fair Allocations of Indivisible Goods." ACM Conference on Electronic Commerce, (2004): 125-131.

Nuchia, S. W. and S. Sen.. "Improving Optimality of N Agent Envy-free Divisions." Proceedings of the 8th International Workshop on Agent Theories, Architecture, and Languages, 2001. 208-217.

Saunders, John H. "A Comparison of Decision Accuracy in the Analytical Hierarchy Process and Point Allocation." www.johnsaunders.com/papers/ahpvpa/ahpvpa.htm. Accessed October 13, 2011.

Srinivas, N. and D. Kalyanmoy. *Multiobjective Optimization Using Nondominated Sorting in Genetic Algorithms*. The Massachusetts Institute of Technology. *Evolutionary Computation* 2 (3), (1994): 221-248.

SolveXL. *SolveXl- Genetic Algorithm Optimization Add-in for Microsoft Excel*. http://www.solvexl.com/. Accessed on July 16, 2013.

Varian, H. "Equity, Envy, and Efficiency." *Journal of Economic Theory* 9, (1974): 63-91.

Vetschera, R. "A General Branch-and-Bound Algorithm for Fair Division Problems." *Computers & Operations Research* 37 (12), (2010): 2121-2130.

Tsunokawa, K. and D. Van Hiep. "A Unified Optimization Procedure for Road Asset Management." *6th ICPT* (2008): 1-8.

Carlos M. Chang, Ph.D., P.E. *Dr. Chang is an assistant professor in the Department of Civil Engineering at the University of Texas at El Paso (UTEP). Dr. Chang's major areas of interest are pavement evaluation and design, pavement management, construction management, knowledge management, infrastructure management, and asset management. He conducts research and teaches courses in his areas of expertise. Among his research sponsors are the National Highway Cooperative Research Program, Metropolitan Transportation Commission in California, the Texas Department of Transportation, and the City of El Paso. Dr. Chang is the academic advisor of the Chi Epsilon Chapter at UTEP and member of the American Society of Civil Engineering. He is recognized as an international consultant and has participated as an expert in major highway projects. He is registered as an international consultant at the Inter-American Development Bank. Dr. Chang is the former chair of the International Road Federation Asset Management Task Force. In 2011, 2012, and 2013, Dr. Chang received the "Engineering Award for Connecting Professional Practice and Education" from the National Council of Examiners for Engineering and Surveying.*

Edith Montes *received a bachelor of science degree in civil engineering from the University of Texas at El Paso (UTEP) in May 2011. She is currently working on her master of science in civil engineering at UTEP. Montes is a graduate research assistant in the Department of Civil Engineering of UTEP. Her thesis topic is "Fair Division Methods for Funding Allocation."*